Advanced Mathematical Models & Applications
Vol.9, No.3, 2024, pp.408-414
https://doi.org/10.62476/amma93408

NUMERICAL SIMULATION OF WAVE DAMPING INDUCED BY VEGETATION ON SOLITARY WAVE RUN-UP WITH MOMENTUM CONSERVATIVE SCHEME

M.A. Ginting[1*], **Y. Dani**[2]

[1,2]Computer Science Department, School of Computer Science, Bina Nusantara University, Bandung Campus, Jakarta, Indonesia

Abstract. In this paper, the damping effects of vegetation are investigated in shallow water model. Wave dissipation due to the inertia and drag force induced by vegetation is implemented based on Morison's formulation. The dissipation term is added in momentum equation. The non-linear model is then solved numerically using momentum conservative scheme on staggered grid domain. To validate the numerical scheme, simulations of solitary wave propagation without vegetation are conducted and compared to the exact solution and laboratory experiment. Besides, several simulations of solitary wave with vegetation on sloping beach are also conducted with varying plant stem density. The numerical results reveal that vegetation significantly reduces the velocity of wave propagation. Additionally, an increase in vegetation stem density correlates with a decrease in solitary wave run-up.

Keywords: Solitary wave, shallow water equations, vegetation damping, numerical simulation.

AMS Subject Classification: 76B25, 65M06.

Corresponding author: Maria Artanta, Ginting, Bina Nusantara University, Jl. K. H. Syahdan No. 9, Kemanggisan, Palmerah, Jakarta, Indonesia, Tel.: +62215345830, e-mail: *maria.ginting@binus.ac.id*

Received: 15 May 2024; Revised: 8 June 2024; Accepted: 21 June 2024; Published: 4 December 2024.

1 Introduction

Coastal areas globally are increasingly threatened by wave-induced hazards, emphasizing the need for precise modeling and prediction of wave behavior. Solitary wave run-up is particularly challenging due to its potential to cause coastal flooding and erosion. Understanding the dynamics of solitary wave run-up and its interaction with coastal vegetation is crucial for effective coastal management and hazard mitigation strategies. Coastal vegetation such as mangroves and salt marshes, as well as belts of sea grass and seaweed has been considered as a natural barrier for coastal zone protection. In recent years, numerical simulations have become powerful tools in studying wave-vegetation interactions and assessing their impact on coastal dynamics (Yin et al., 2023; Zhang et al., 2022; van Veelen et al., 2020; Lou et al., 2018).

The damping of water waves typically arises from energy dissipation due to the work exerted on plants. Various numerical and analytical models have been developed to relate the interactions between waves and vegetative plants, aiming to explain the damping effects of vegetation. In many studies focusing on wave-vegetation interactions, solitary waves are commonly employed to represent extreme phenomena like tsunami. Huang et al. (2011) examined the drag

How to cite (APA): Ginting, M.A., Dani, Y. (2024). Numerical simulation of wave damping induced by vegetation on solitary wave run-up with momentum conservative scheme. *Advanced Mathematical Models & Applications*, *9*(3), 408-414 https://doi.org/10.62476/amma93408

coefficient of vegetation in response to solitary wave interaction, exploring various arrangements and widths of vegetation belts. Tang et al. (2013) conducted numerical investigations into solitary wave damping induced by vegetation. Subsequently, they extended the research to include the interaction of solitary wave and vegetation on sloping beach (Tang et al. (2015)) and studied the run-up of long-period waves with rigid vegetation (Tang et al., 2017). Tognin et al. (2019) highlighted the significance of vegetation density, velocity profile variations, and flow visualizations in understanding the passage of solitary waves over vegetation belts. Divya et al. (2020) conducted a numerical investigation, treating vegetation patches as varying porous layers within a particle-based method. Similarly, Zhao et al. (2020) numerically examined the reduction in wave run-up provided by vegetation, exploring various configurations of vegetation patches for solitary waves. Zhang and Nepf (2021) studied the existence of current impacted wave dissipation in a meadow of flexible marsh plants. They developed a wave-damping model from a prediction of current- and wave-induced force on individual plants. Furthermore, wave damping due to flexible vegetation has been investigated by Ding et al. (2022). They simulated wave attenuation and mean water level changes through flexible vegetations such as smooth cordgrass in coastal and marine wetlands numerically. The analytical solutions of wave attenuation by flexible vegetations was investigated by Zhu et al. (2022) .

This paper focuses on investigating wave damping induced by vegetation on solitary wave run-up through numerical simulations. We implemented a momentum conservative scheme on shallow water equations. The numerical scheme was proposed by Stelling & Zijlema (2003). Several standardized numerical tests have demonstrated the robustness and the accuracy of the scheme (Stelling & Duinmeijer, 2003; Nguyen, 2013; Herbin et al., 2014; Zheng et al., 2023; Aliev et al., 2012). In addition, the staggered grid scheme is simple and easy for many applications such as shallow water viscous water equations. The dissipation effect induced by the presence of the vegetation is modeled by incorporating Morison's formula (Morison et al., 1950) in momentum equation. The accuracy of the numerical model is validated against a laboratory experiment on solitary wave propagation conducted by Synolakis (1987). First, the model is tested for solitary wave run-up without vegetation to asses the accuracy of the scheme. Later, series of simulations of solitary wave run-up with vegetation on sloping beach is conducted to investigate the sensitivity of the wave to stem density.

This paper is organized as follows. Section 2 presents the wave model of shallow water equations and the numerical scheme. In Section 3, we discuss the numerical simulation results of solitary wave run-up. Finally, the conclusion is presented in Section 4.

2 Wave Model and Numerical Scheme

In this section, we delve into the governing equations, i.e. the shallow water equations (SWE), which are derived from Euler equations. Consider a layer of inviscid fluid with zero thermal conductivity, with a free surface $z = \eta(x,t)$ and topography $z = -d(x)$. The shallow water model are expressed as follows.

$$h_t + (hu)_x = 0 \tag{1}$$

$$u_t + uu_x + g\eta_x + \frac{1}{\rho}\tau_f + \frac{1}{\rho}\tau_v = 0. \tag{2}$$

The first equation is conservation of mass (continuity), whereas the subsequent one is balance of momentum. The variables in these equations are illustrated in Figure 1. Here $u(x,t)$ denotes the fluid particle velocity in horizontal direction, g is the gravitational acceleration, and the total water thickness is denoted by $h(x,t) = \eta(x,t) + d(x)$. In shallow water equations, the horizontal domain is assumed to be much longer than the vertical domain. The horizontal velocity is calculated in terms of its depth averaged. In momentum equation, we take into account the

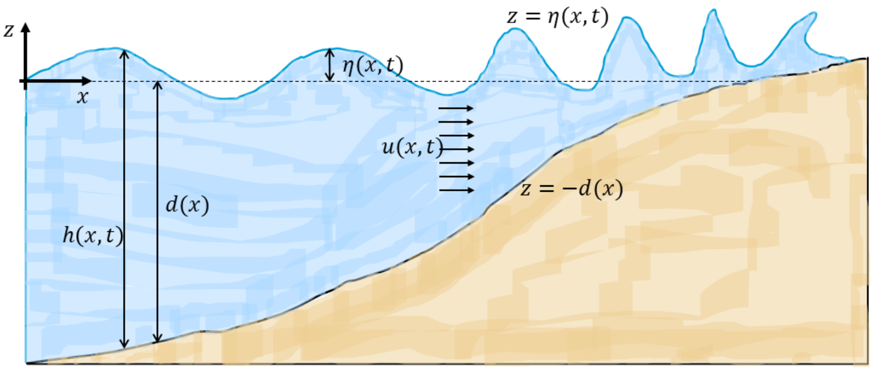

Figure 1: Position of variables in shallow water model

dissipation terms. The friction force due to the bottom roughness is denoted by τ_f, and the damping due to vegetation is denoted by τ_v, where ρ is the water density.

Attenuation due to vegetation consists of drag and inertia forces using the Morison's formulation (1950) as

$$\tau_v = \frac{1}{2}\rho c_d b_v h_v N_v u|u| + c_{in}(hu)_t, \tag{3}$$

where c_d and c_{in} are drag and inertia coefficients defined as (Harada & Imamura (2005))

$$c_d = 0.10 + 0.8 r_V, \quad (0.01 \le r_V \le 0.07) \tag{4}$$

$$c_{in} = c_m b_v d_v h_v N_v, \quad (c_m = 0.15) \tag{5}$$

and b_v is width of a stem normal to wave, h_v is stem height, N_v is density of stem (number of stem per unit area), d_v is stem diameter, r_V is ratio of vegetation volume under water to water volume.

Equation (1)-(2) are solved numerically using momentum conservative scheme in staggered grid domain (Stelling & Zijlema (2003)). In this method, the horizontal domain $x \in [0, L]$ is discretized by

$$0 = x_{\frac{1}{2}},\ x_1,\ x_{\frac{3}{2}},\ x_2,\ \ldots,\ x_N, x_{N+\frac{1}{2}} = L. \tag{6}$$

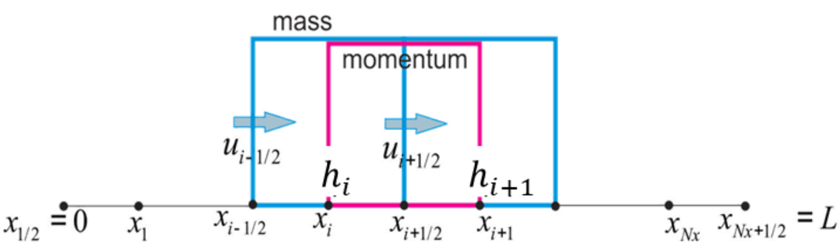

Figure 2: Unknowns calculated in staggered grid domain

Position of variables in staggered grid is illustrated in Figure 2. The horizontal velocity u is calculated at half point $x_{i-\frac{1}{2}}$ and time t^n denoted as $u^n_{i-\frac{1}{2}}$, while water height h and wave elevation η are calculated at full point x_i and time t^n respectively denoted as h^n_i and η^n_i, for

$i = 1...N+1$. Therefore, the discrete equations of shallow water model can be written as follows

$$\frac{h_i^{n+1} - h_i^n}{\Delta t} + \frac{{}^*h_{i+\frac{1}{2}}^n u_{i+\frac{1}{2}}^n - {}^*h_{i-\frac{1}{2}}^n u_{i-\frac{1}{2}}^n}{\Delta x} = 0, \tag{7}$$

$$\frac{u_{i+\frac{1}{2}}^{n+1} - u_{i+\frac{1}{2}}^n}{\Delta t} + (uu_x)_{i+\frac{1}{2}}^n + g\left(\frac{\eta_{i+1}^n - \eta_i^n}{\Delta x}\right) + \frac{1}{\rho}\tau_f + \frac{1}{\rho}(\tau_v)_{i+\frac{1}{2}}^n = 0. \tag{8}$$

In equation (7), the notation $^*h$ is an approximation of water height at half points, using upwind method that follows the water flow direction, which is defined by

$$^*h_{i+\frac{1}{2}}^n = \begin{cases} h_i^n & \text{if } u_{1,i+\frac{1}{2}}^n > 0 \\ h_{i+1}^n & \text{if } u_{i+\frac{1}{2}}^n < 0 \\ \max(\eta_i^n, \eta_{i+1}^n) + \min(d_i, d_{i+1}) & \text{if } u_{i+\frac{1}{2}}^n = 0. \end{cases} \tag{9}$$

The momentum equation (8) is calculated when the cell $[x_{i-1}, x_i]$ is wet, with condition $h_{i-1} > 0$ and $h_i > 0$. Here, the advection term are approximated using momentum conservative scheme which is derived in Stelling & Busnelli (2001), as follows

$$(uu_x)_{i-\frac{1}{2}} = \frac{1}{\bar{h}_{1-\frac{1}{2}}}\left(\frac{\bar{q}_i u_i^* - \bar{q}_i u_{i-1}^*}{\Delta x} - u_{i-\frac{1}{2}}\frac{\bar{q}_i - \bar{q}_{i-1}}{\Delta x}\right) \tag{10}$$

where

$$\bar{h}_{i-\frac{1}{2}} = \frac{1}{2}(h_{i-1} + h_i) \tag{11}$$

$$\bar{q}_i = \frac{1}{2}\left(q_{i+\frac{1}{2}} - q_{i-\frac{1}{2}}\right) \tag{12}$$

$$q_{i-\frac{1}{2}} = {}^*h_{i-\frac{1}{2}}\bar{u}_{i-\frac{1}{2}} \tag{13}$$

$$u_i^* = \begin{cases} u_{i-\frac{1}{2}} & \text{if } \bar{q}_i \geq 0 \\ u_{i+\frac{1}{2}} & \text{if } \bar{q}_i < 0. \end{cases} \tag{14}$$

3 Numerical Results and Discussion

In this section, we evaluate the performance of the numerical scheme implemented in shallow water model incorporating vegetation. Numerical simulations are conducted to demonstrate propagation of solitary waves. Additionally, we aim to investigate the wave attenuation resulting from vegetation.

3.1 Solitary Wave Run-up without Vegetation on Sloping Beach

The simulation of solitary wave run-up without vegetation aims to validate the accuracy of the numerical scheme. Consider the following solitary wave at time $t = 0$ and a peak at $x = X_1$,

$$\eta(x, 0) = H\text{sech}^2\gamma(x - X_1) \tag{15}$$

where $\gamma = \sqrt{3H/4}$ and H is the normalized wave height. The simulation was conducted based on the laboratory experiment by Synolakis (1987). The topography is a plane beach with $1 : 19.85$ slope. We simulated a non-breaking wave case with wave height $H = 0.0185$. The computational parameters we took were $\Delta x = 0.2$ and $\Delta t = 0.01$ as discretization in horizontal grid and in time, respectively. Results of the numerical simulation compared to the experimental data are presented in Figure 3; it shows the wave profiles are at four dimensionless time, (a) $t = 40$, (b) $t = 50$, (c) $t = 60$, and (d) $t = 70$. The simulated results align well with the laboratory data, indicating that the numerical model effectively predicts the run-up of non-breaking solitary waves on the beach.

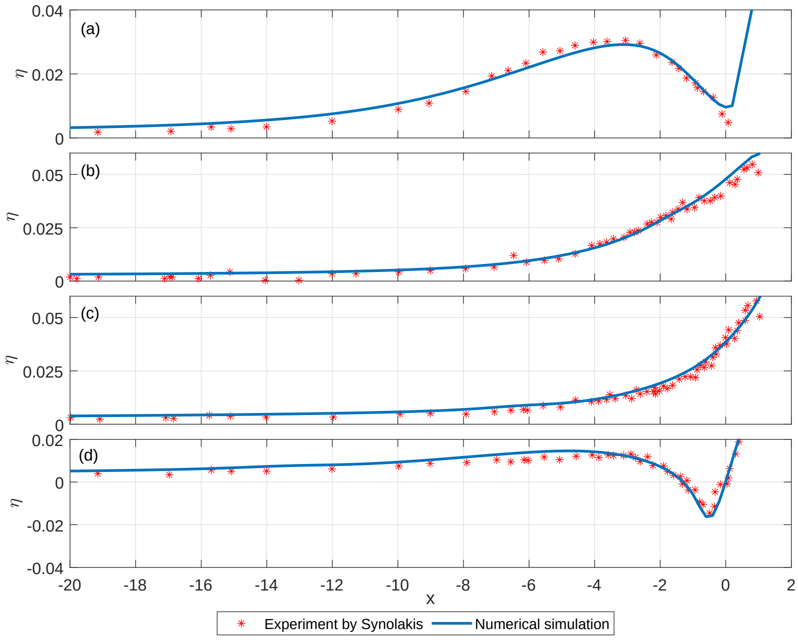

Figure 3: Solitary wave run-up from experiment (red) and numerical simulation (blue).

3.2 Solitary Wave Run-up with Vegetation on Sloping Beach

In this simulation, we demonstrate propagation of solitary wave on a sloping beach featuring vegetation and bottom friction. We also aim to investigate the sensitivity of solitary wave run-up to stem density by conducting comparisons among four cases with varying plant stem densities.

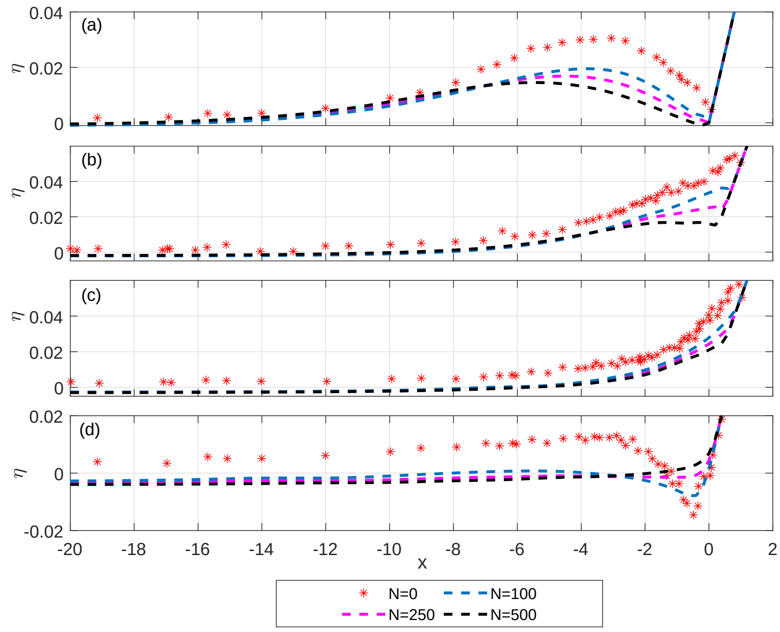

Figure 4: Solitary wave run-up with vegetation damping

The simulation was carried out with initial solitary wave as in equation (15). The topography is a plane beach with 1 : 19.85 slope, and plants are placed from the toe of the beach to the sloping area with a length of 20. We employ identical computational parameters as those utilized in the previous test; $H = 0.0185$, $\Delta x = 0.2$ and $\Delta t = 0.01$. In simulations involving wave attenuation, the bottom friction is set to $\tau_f = 0.001$, the vegetation parameters are $h_v = 1.2$, $d_v = 0.025$, and $N_v = 0, 100, 250, 500$.

The numerical simulation results are presented in Figure 4 for subsequent dimensionless time, (a) $t = 40$, (b) $t = 50$, (c) $t = 60$, and (d) $t = 70$ and four varying stem density. We can see from the figure that in the case without vegetation ($N_v = 0$), the solitary wave crest and run-up travel faster and higher compared to those in cases with vegetation. Moreover, the solitary wave crest and run-up in the case with $N_v = 500$ travel at the slowest pace and reach the lowest height among all cases. This indicates that solitary wave propagation through vegetation is slower than in the absence of vegetation, with the propagation velocity decreasing as the plant stem density increases. Additionally, it is observable that solitary wave run-up in vegetation is lower than that without vegetation, and the run-up decreases as the plant stem density increases.

4 Conclusion

Numerical simulations have been conducted to investigate the damping effects of vegetation on solitary water wave run-up. The model incorporated Morison's equation to account for vegetation-induced inertia and drag stresses within the nonlinear shallow water equations. The model was solved numerically using momentum conservative scheme on a staggered grid. The numerical scheme's accuracy was confirmed through validation with experimental data before being utilized to observe the sensitivity of solitary wave run-up to vegetation. The results indicate that vegetation significantly reduces propagation velocity and run-up height, with solitary wave run-up decreasing as stem density increases.

References

Aliev, F.A., Gasimov, Y.S., Velieva, N.I., Safarova, N.A., & Agamalieva, L.F. (2012). High accuracy algorithms to the solution of the optimal output feedback problem for the linear systems. *Proceedings of the Romanian Academy, Series A, 13*(3), 207-214.

Ding, Y., Rosati, J.D., Johnson, B.D., Chen, Q.J., & Zhu, L. (2022). Implementation of flexible vegetation into CSHORE for modeling wave attenuation. Flood and Coastal Systems Program.

Divya, R., Sriram, V., & Murali, K. (2020). Wave-vegetation interaction using improved meshless local Petrov Galerkin method. *Applied Ocean Research*, *101*, 102116.

Harada, K., Imamura, F. (2005). Effects of coastal forest on tsunami hazard mitigation-a preliminary investigation. *Advances in natural and technological hazards research*, *23*, 279.

Herbin, R., Kheriji, W., & Latché, J.C. (2014). On some implicit and semi-implicit staggered schemes for the shallow water and Euler equations. *ESAIM: Mathematical Modelling and Numerical Analysis*, *48*(6), 1807-1857.

Huang, Z., Yao, Y., Sim, S. Y., & Yao, Y. (2011). Interaction of solitary waves with emergent, rigid vegetation. *Ocean Engineering*, *38*(10), 1080-1088.

Lou, S., Chen, M., Ma, G., Liu, S., & Zhong, G. (2018). Laboratory study of the effect of vertically varying vegetation density on waves, currents and wave-current interactions. *Applied Ocean Research*, *79*, 74-87.

Ma, Y., Zhu, L., Peng, Z., Xue, L., Zhao, W., Li, T., ... & Li, X. (2023). Wave attenuation by flattened vegetation (Scirpus mariqueter). *Frontiers in Marine Science*, *10*, 1106070.

Morison, J. R., Johnson, J. W., & Schaaf, S. A. (1950). The force exerted by surface waves on piles. *Journal of Petroleum Technology*, *2*(05), 149-154.

Nguyen, T.T. (2013). *Explicit staggered schemes for compressible flows.* (Doctoral dissertation, Aix-Marseille université).

Stelling, G.S., Busnelli, M.M. (2001). Numerical simulation of the vertical structure of discontinuous flows. *International Journal for Numerical Methods in Fluids*, *37*(1), 23-43.

Stelling, G.S., Duinmeijer, S.A. (2003). A staggered conservative scheme for every Froude number in rapidly varied shallow water flows. *International Journal for Numerical Methods in Fluids*, *43*(12), 1329-1354.

Stelling, G., Zijlema, M. (2003). An accurate and efficient finite-difference algorithm for non-hydrostatic free-surface flow with application to wave propagation. *International Journal for Numerical Methods in Fluids*, *43*(1), 1-23.

Synolakis, C.E. (1987). The runup of solitary waves. *Journal of Fluid Mechanics*, *185*, 523-545.

Tang, J., Causon, D., Mingham, C., & Qian, L. (2013). Numerical study of vegetation damping effects on solitary wave run-up using the nonlinear shallow water equations. *Coastal Engineering*, *75*, 21-28.

Tang, J., Shen, S., & Wang, H. (2015). Numerical model for coastal wave propagation through mild slope zone in the presence of rigid vegetation. *Coastal Engineering*, *97*, 53-59.

Tang, J., Shen, Y., Causon, D.M., Qian, L., & Mingham, C.G. (2017). Numerical study of periodic long wave run-up on a rigid vegetation sloping beach. *Coastal Engineering*, *121*, 158-166.

Tognin, D., Peruzzo, P., De Serio, F., Ben Meftah, M., Carniello, L., Defina, A., & Mossa, M. (2019). Experimental setup and measuring system to study solitary wave interaction with rigid emergent vegetation. *Sensors*, *19*(8), 1787.

van Veelen, T.J., Fairchild, T.P., Reeve, D.E., & Karunarathna, H. (2020). Experimental study on vegetation flexibility as control parameter for wave damping and velocity structure. *Coastal Engineering*, *157*, 103648.

Yin, K., Xu, S., Huang, W., & Lin, M. (2023). Experimental and numerical investigation of wave-induced dynamics of emergent flexible vegetation. *Ocean Engineering*, 289, 116155.

Zhang, W., Ge, Z.M., Li, S.H., Tan, L.S., Zhou, K., Li, Y.L., ... & Dai, Z.J. (2022). The role of seasonal vegetation properties in determining the wave attenuation capacity of coastal marshes: Implications for building natural defenses. *Ecological Engineering*, *175*, 106494.

Zhang, X., Nepf, H. (2021). Wave damping by flexible marsh plants influenced by current. *Physical Review Fluids*, *6*(10), 100502.

Zhao, C., Zhang, Y., Tang, J., & Shen, Y. (2020). Numerical investigation of solitary wave run-up attenuation by patchy vegetation. *Acta Oceanologica Sinica*, *39*, 105-114.

Zheng, Y., Cheng, F., Liu, J., Fan, Z., Han, B., & Wang, J. (2023). Elastic full-wave field simulation in 3D tunnel space with the variable staggered-grid finite-difference method in cylindrical coordinates. *Journal of Applied Geophysics*, 213, 105013.

Zhu, L., Huguenard, K., Fredriksson, D. W., & Lei, J. (2022). Wave attenuation by flexible vegetation (and suspended kelp) with blade motion: Analytical solutions. *Advances in Water Resources*, *162*, 104148.